The study of time series of monthly averaged values of $F_{10.7}$ from 1950 to 2010

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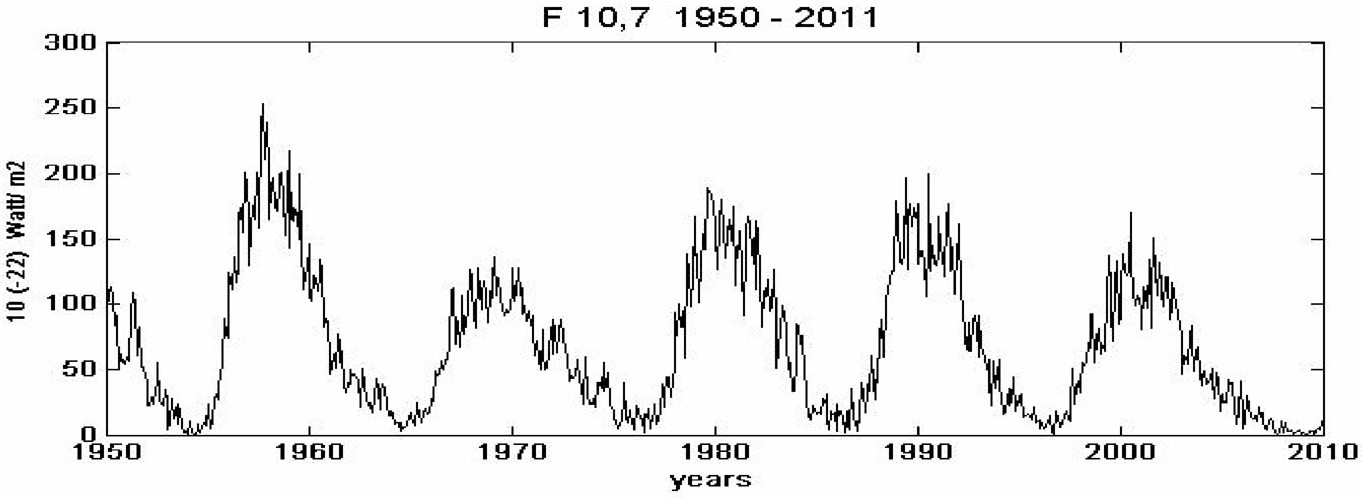
- **The solar radio microwave flux at wavelengths 10.7 cm $F_{10.7}$ has the long running series of observations started in 1947 in Ottawa, Canada and maintained to this day at Penticton site in British Columbia.**
- **This radio emission comes from high part of the chromosphere and low part of the corona.**
- **$F_{10,7}$ radio flux has two different sources: thermal bremsstrahlung (due to electrons radiating when changing direction by being deflected by other charged participles) and gyro-radiation (due to electrons radiating when changing direction by gyrating around magnetic fields lines).**
- **These mechanisms give rise to enhanced radiation when the temperature, density and magnetic fields are enhanced. So $F_{10,7}$ is a good measure of general solar activity.**
- **$F_{10.7}$ data are available at**

http://radbelts.gsfc.nasa.gov/RB_model_int/Psi_database.html

The time series of monthly average of $F_{10.7}$ from 1950 to 2010. National Geophysical Data Center of Solar and Terrestrial Physics data.



We have made the analysis of time series of $F_{10.7}$ with the use of different mother wavelets: Daubechies 10, Simlet 8, Meyer, Gauss 8 and Morlet (real and complex).

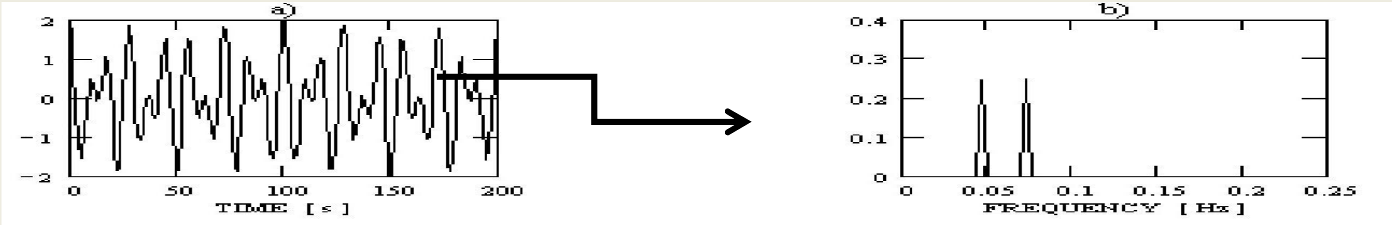
$$\tilde{f}(\nu) = \int_{-\infty}^{\infty} f(t) e^{-i2\pi\nu t}\, dt$$

$$f(t) = A\cos(2\pi\nu_0 t)$$

$$f(t) = \int_{-\infty}^{\infty} \tilde{f}(\nu) e^{i2\pi\nu t}\, d\nu$$

$$\tilde{f}(\nu) = A\left[\delta(\nu - \nu_0) + \delta(\nu + \nu_0)\right]$$

Fourier - transformation



Fourier analysis consists of breaking up a signal into sine waves of various frequencies.

Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or mother) wavelet.

- **The wavelets bring their own strong benefits to that environment: a local outlook, a multiscaled outlook, cooperation between scales, and a time-scale analysis.**
- **They demonstrate that sines and cosines are not the only useful functions and that other bases made of weird functions serve to look at new foreign signals, as strange as most fractals or some transient signals.**
- **The wavelets are the localized functions constructed with help**

of one so-called mother wavelet $\psi(t)$

by shift operation on argument (b) : $\psi_{ab}(t) = (1/\sqrt{|a|}) \cdot \psi((t-b)/a)$

and scale change (a) : $\psi((t-b)/a)$

- **The wavelet time-scale spectrum C(a,b) is the two-arguments function**

'a' is measured in

reversed-frequency units

'b' is measured in time units: $C(a,b) = (1/\sqrt{|a|}) \int_{-\infty}^{\infty} s(t) \cdot \psi((t-b)/a)\,dt$

The choice of wavelet is dictated by the signal or image characteristics and the nature of the application. If you understand the properties of the analysis and synthesis wavelet, you can choose a wavelet that is optimized for your application.

Wavelet families vary in terms of several important properties. Examples include:

• Support of the wavelet in time and frequency and rate of decay.

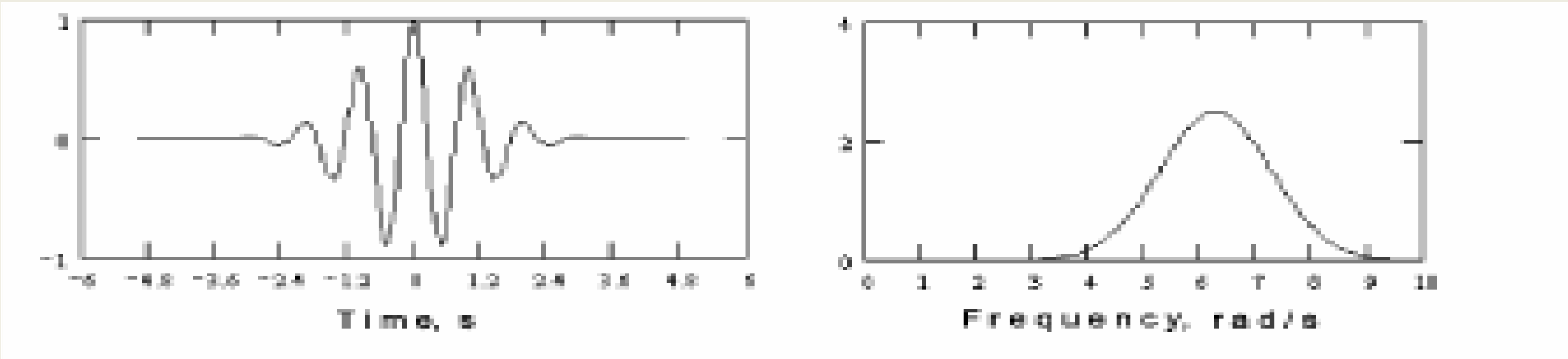
• Symmetry or antisymmetry of the wavelet. The accompanying perfect reconstruction filters have linear phase.

•Number of vanishing moments. Wavelets with increasing numbers of vanishing moments result in sparse representations for a large class of signals and images.

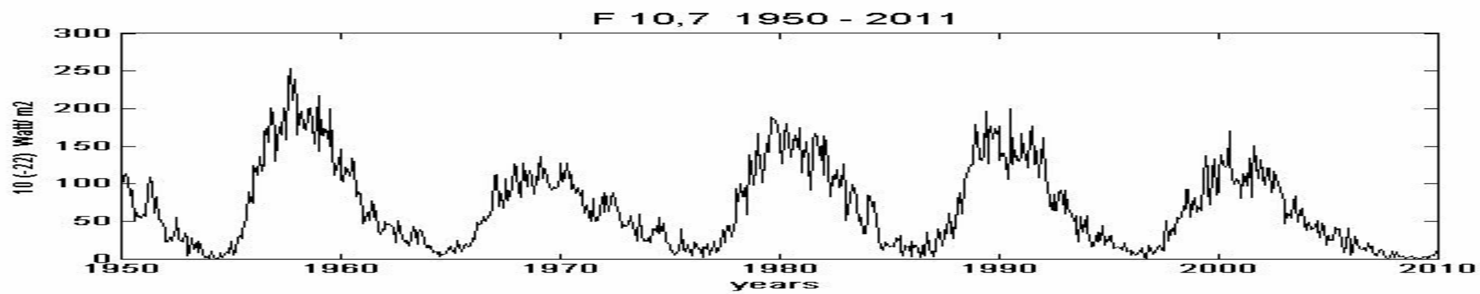
• Regularity of the wavelet. Smoother wavelets provide sharper frequency resolution. Additionally, iterative algorithms for wavelet construction converge faster.

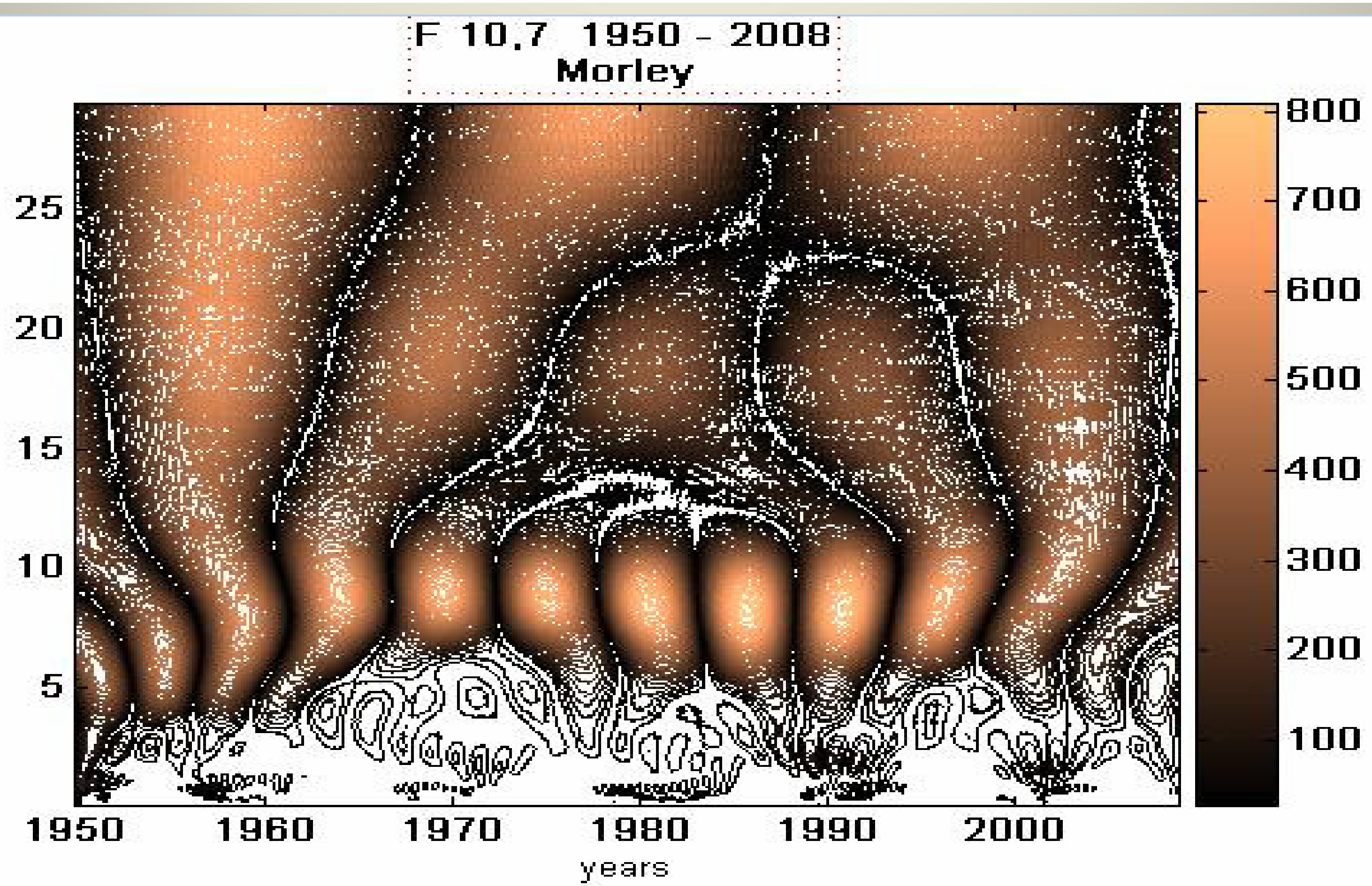
The Morlet wavelet – is the plane wave, modulated by Gaussian function

$$\psi(t) = e^{-\frac{t^2}{\alpha^2}} e^{i 2 \pi t}$$



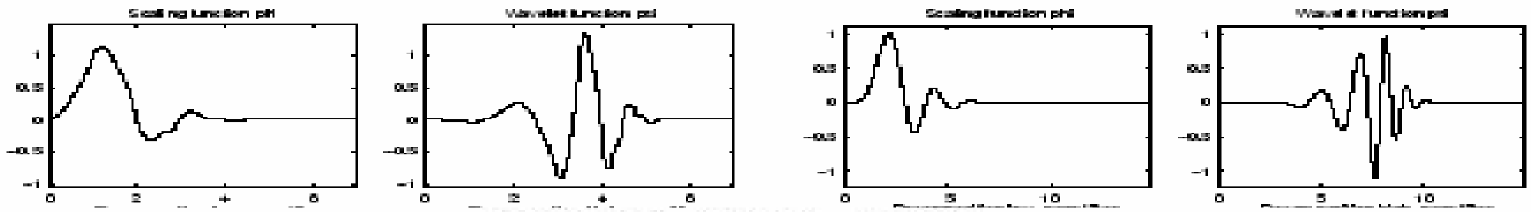
The Morlet wavelet (left) and its Fourier -transformation (right)

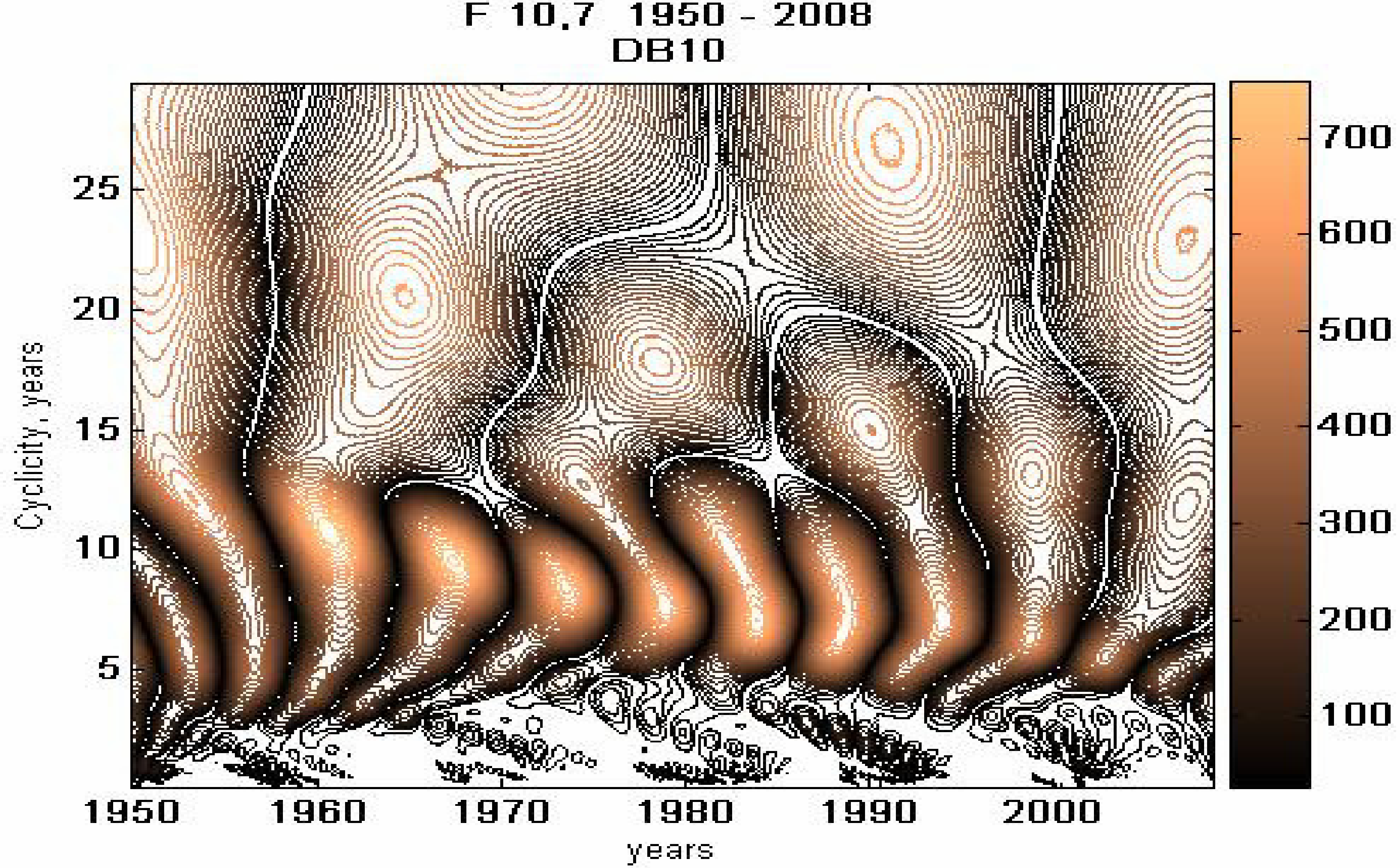




Analysis of time series of $F_{10.7}$ with the use of Morlet mother wavelet

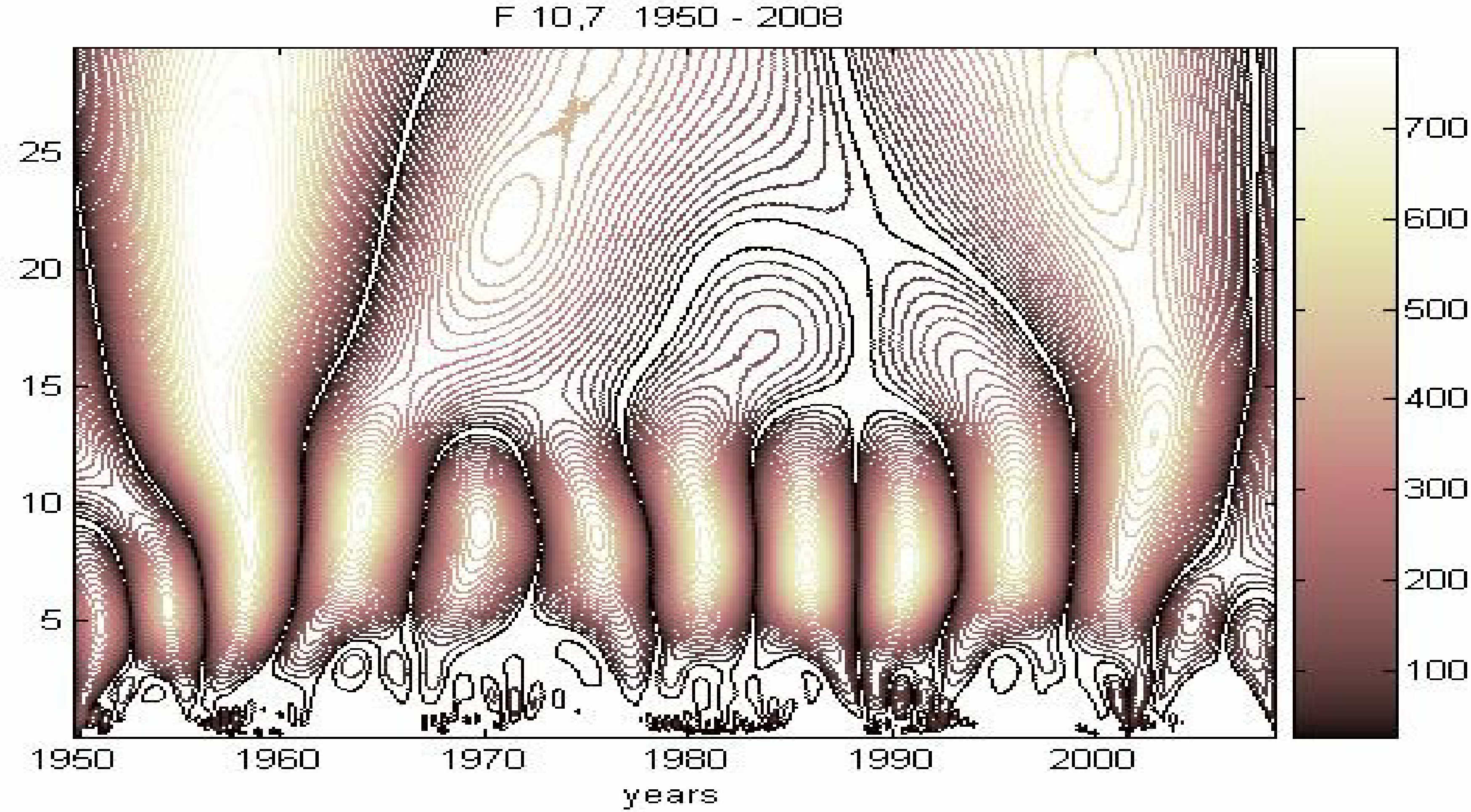
Daubechies' wavelet from 4 to 8 vanishin moments



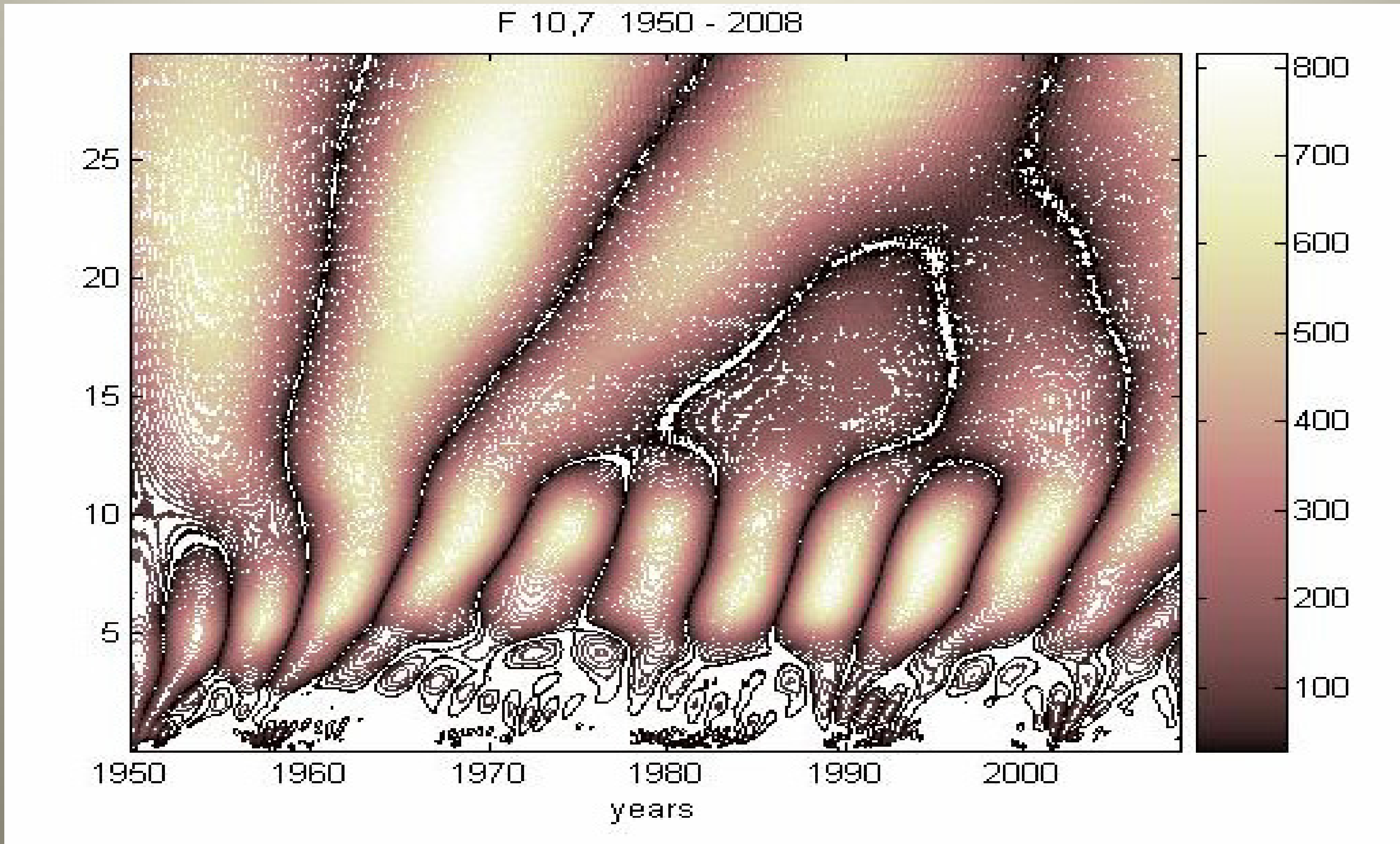


Analysis of time series of $F_{10.7}$ with the use of Daubechies 10 mother wavelet

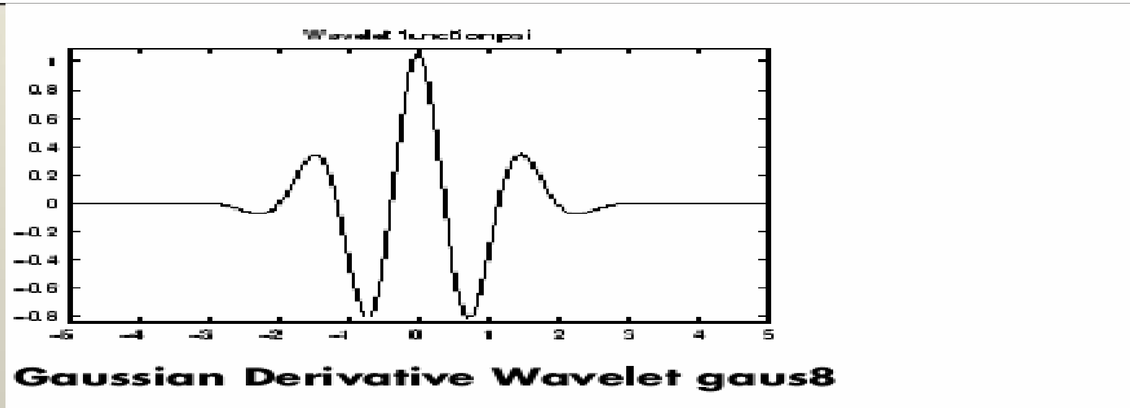
Simlet wavelets from 4 to 8 vanishin moments

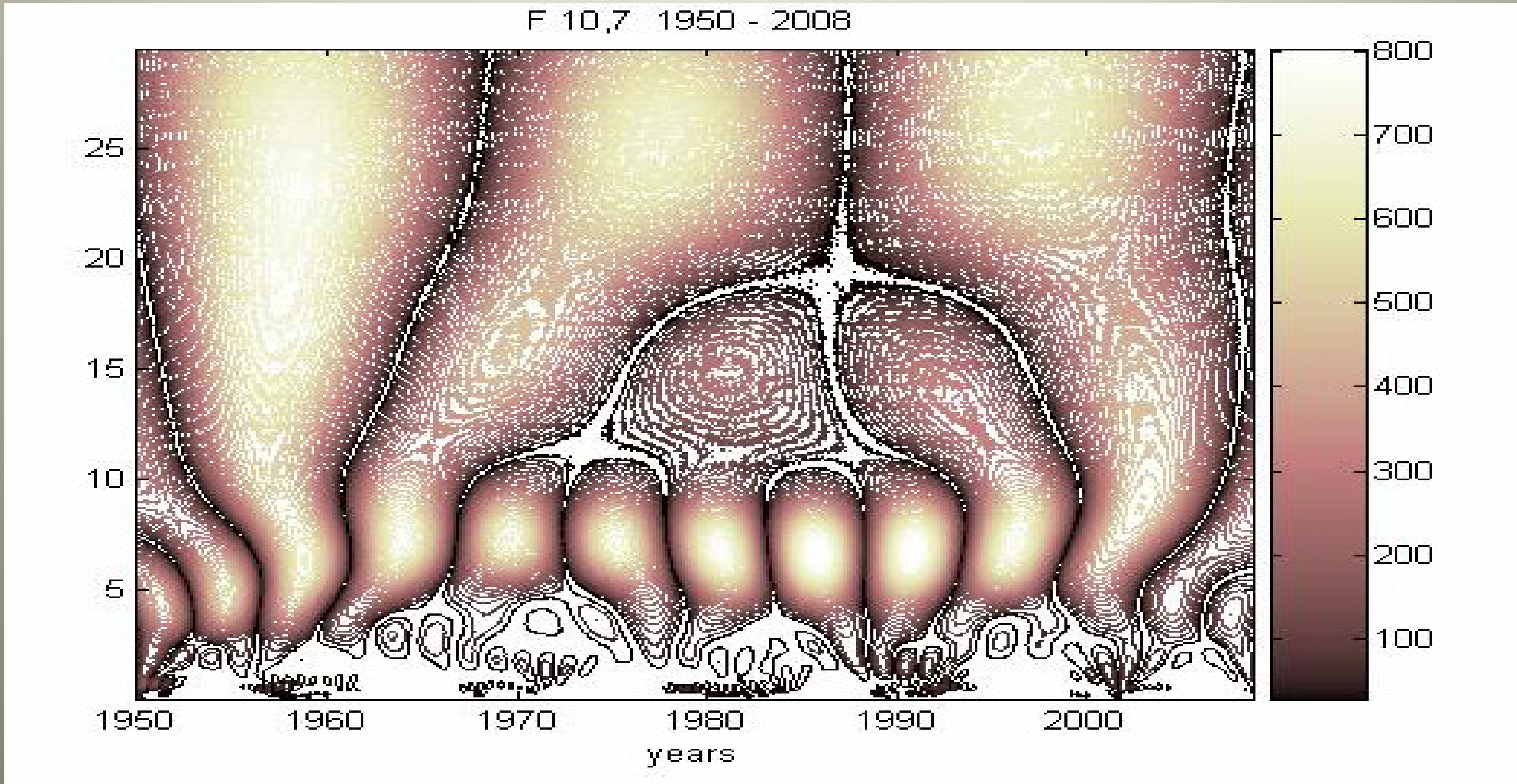


Analysis of time series of $F_{10.7}$ with the use of Simlet 8 mother wavelet



Analysis of time series of $F_{10,7}$ with the use of Meyer mother wavelet





Analysis of time series of $F_{10.7}$ with the use of Gauss 8 mother wavelet

Complex Morlet Wavelets: cmor

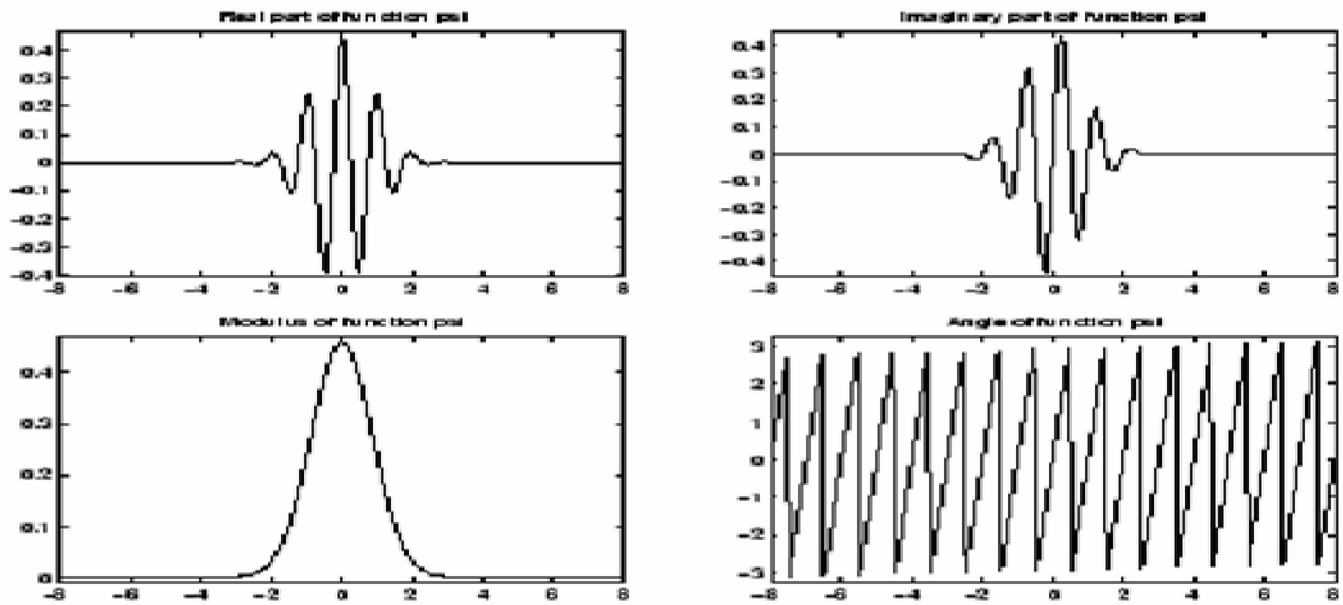
A Complex Morlet Wavelets is defined by

$$\psi(x) = \frac{1}{\sqrt{\pi f_b}} e^{2i\pi f_c x} e^{\frac{x^2}{f^b}}$$

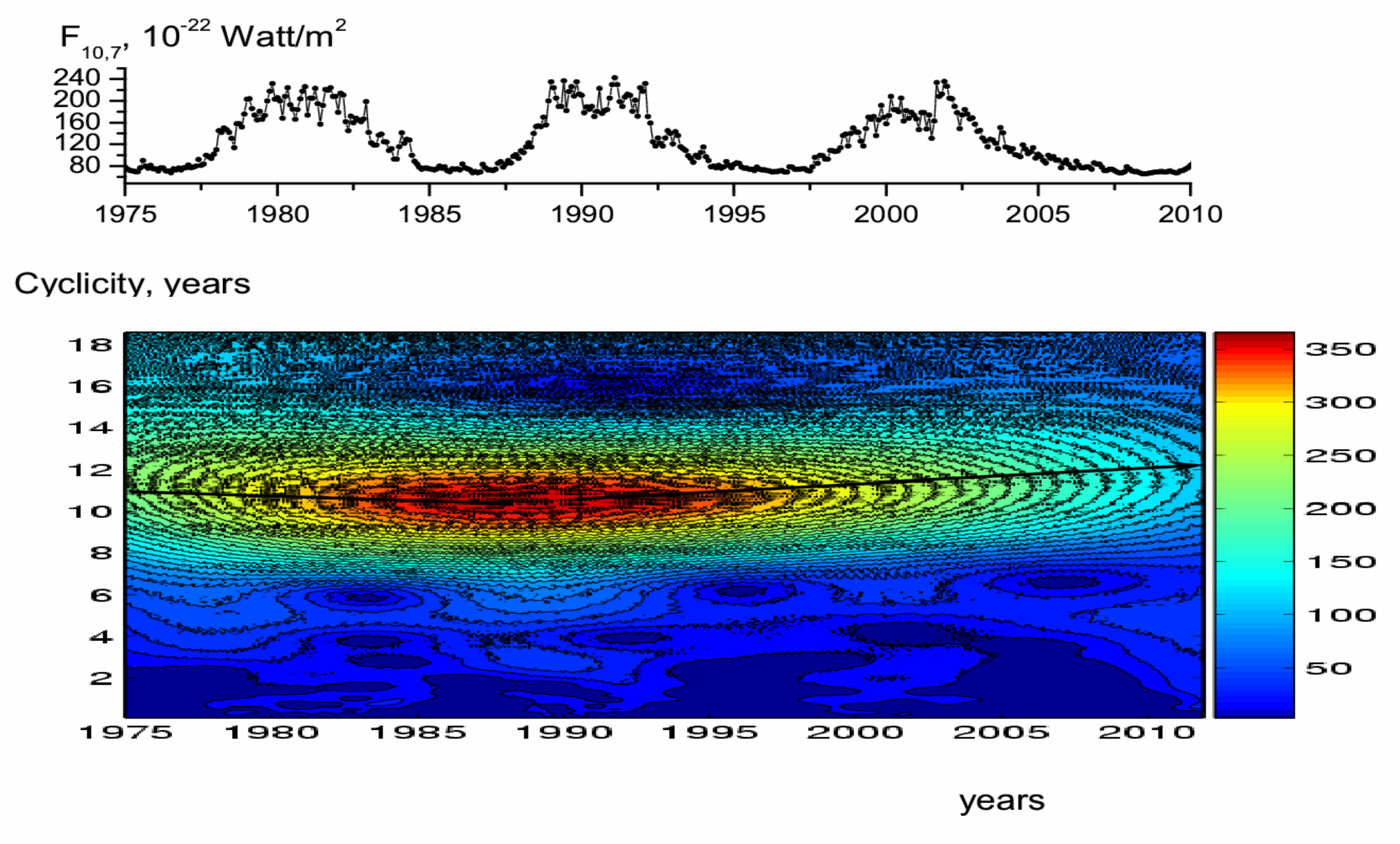
Depending on two parametrs:

- f_b is a bandwidth parameter
- f_e is a wavelet center frequency

You can obtain a survey of the main properties of this family by typing waneinfo ('cmor') from the MATLAB command line



Complex Morlet Wavelet morl 1.5-1



Analysis of time series of $F_{10.7}$ with the use of Complex Morley mother wavelet. Note that in this analysis the main cycle (11-year) is dominated and other cycles with lower amplitudes are suppressed

Wavelet spectrum allows us not only to identify cycles, but analyze their change in time.

Each wavelet has its own characteristic features, so sometimes with the help of different wavelets it can be better identify and highlight the different properties of the analyzed signal.

We intended to choose the mother wavelet, which is more fully gives information about the analyzed solar index $F_{10.7}$.
These are Morley and Gauss real-valued and complex-valued wavelets. With these wavelets we can use the solar cyclicity evolution of the most accurate form in every moment of time. We see also a number of periods, which, perhaps, are the harmonics of main period The complex-valued Morley and Gauss wavelet analysis gives us the additional information about signal phase evolution.
The Daubechies 10 wavelet-analysis (more wide filter window allow us to analyze the influence of the previous cycle to the next).
The Mexican hat wavelet inhibits the main cyclicity and allows us to analyzed the cyclicity of second- order.

The analysis with all mother wavelet shows that the mean value of 11-year cycle is about 10.2 years during the period 1950 - 2010.

Thank You